An incremental Tensor - Train decomposition for high-dimensional data streams

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Why do we care?

- Tensors are representations for multivariate or high-dimensional data.
- Storage and computational requirements scale exponentially with dimensions as the size of a tensor increases.
- Tensor decompositions provide compressed representation of tensors.

What is a tensor-train?

Tensor-train[1] (TT) is a compact representation format for n dimensional tensors.

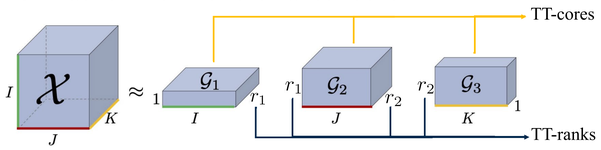

Figure 1: Visual representation of a 3-dimensional tensor in TT-format [2]

Rank of the mode-i unfolding of the d-way tensor $\mathcal{X} \in \mathbb{R}^{n_1 \times \cdots \times n_d}$ provides an upper bound to the i-th TT-rank r_i and is computed as

$$X_{(i)} = \texttt{reshape}(\mathcal{X}[n_i, n_1 \dots n_{i-1} n_{i+1} \dots n_d])$$

The i-th TT-core $\mathcal{G}_i$ can be interpreted as basis for the i-th dimension if reshaped as

$$U_i = \texttt{reshape}(\mathcal{G}_i, [r_{i-1} n_i, r_i])$$

Why do we need incremental algorithms?

In many applications, data is streaming. Even when it is available, the size of data becomes a bottleneck due to limited memory.

Existing incremental algorithms such as ITTD[3] and TT-FOA[4] have limitations:

- ✕ Inefficient rank adaptation or fixed rank with no adaptivity
- ✕ No compression guarantees

What do we offer?

- ✓ **Controlled rank growth**
- ✓ **Obtaining and maintaining target accuracy**
- ✓ **Low memory footprint**
- ✓ **Faster execution**

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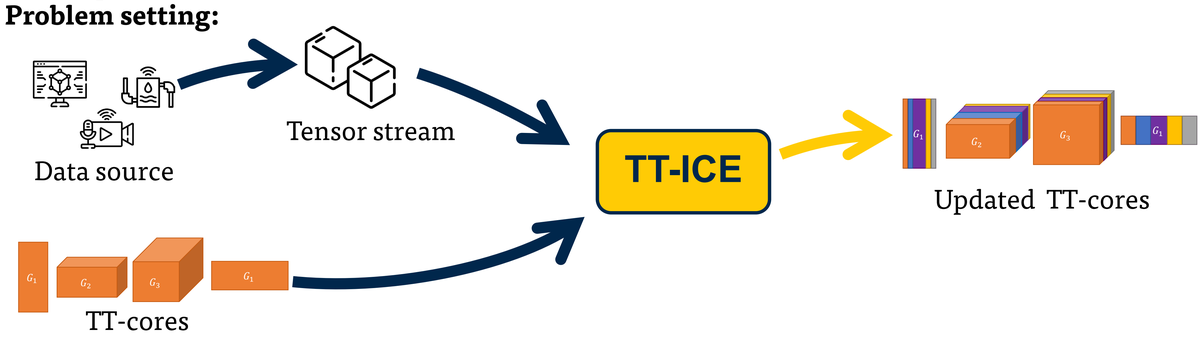
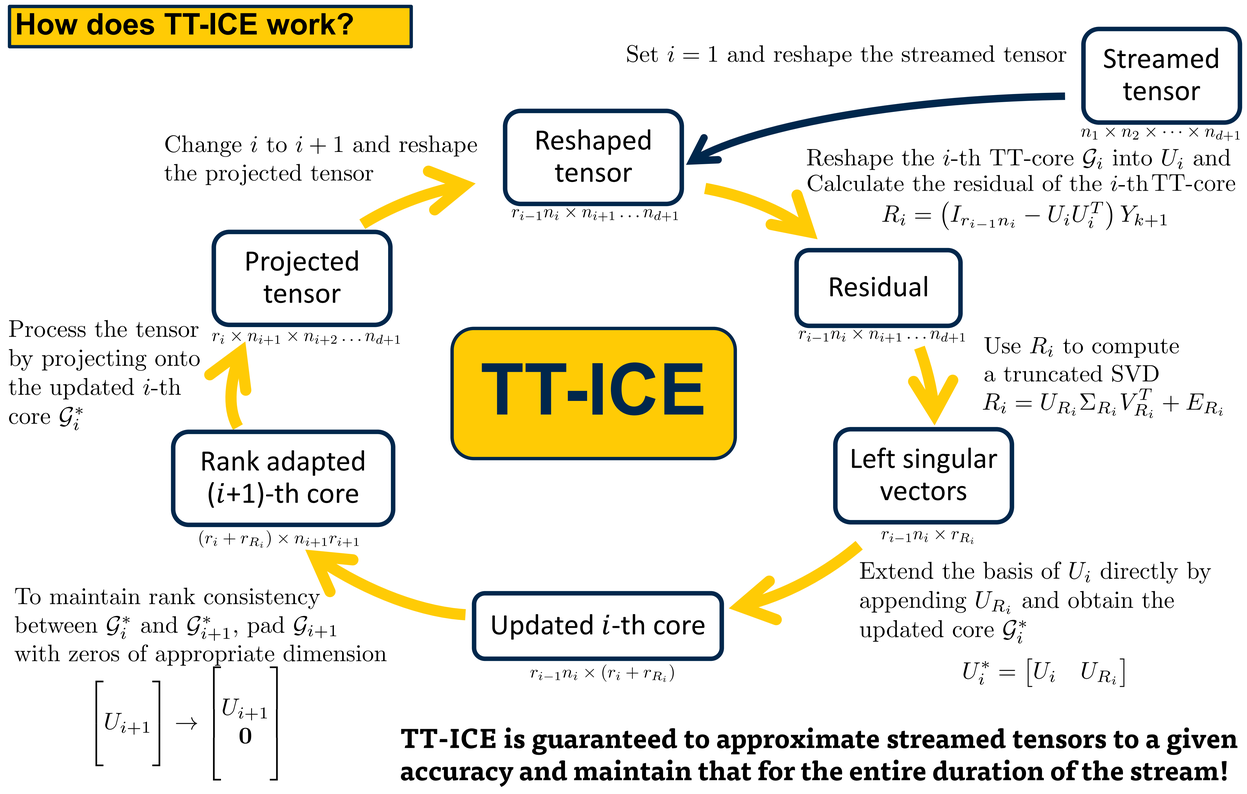
How do we achieve 57× higher compression in 5% of time?[5]

Main idea: Each core represents a basis for the corresponding dimension. We expand or adapt this basis when it is insufficient for representing a new data point. We call this algorithm TT-Incremental Core Expansion (TT-ICE)[5].

Problem setting:



How does TT-ICE work?



TT-ICE is guaranteed to approximate streamed tensors to a given accuracy and maintain that for the entire duration of the stream!

How do we make it faster?

We have developed three heuristics to further speed up the process. This heuristically enhanced version of TT-ICE is called TT-ICE*[5].

Skip updating all cores

When the existing basis adequately represents a new tensor, we don't update them.

Select a subset from batch

If multiple tensors arrive in a batch, we only update with those that are not well represented.

Skip updating filled cores

If a core is full rank, skip updating that core and proceed with the next one.

How does it perform?

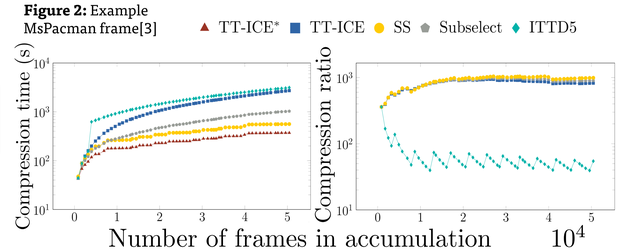
We benchmark TT-ICE and TT-ICE* against ITTD [4], the only available algorithm with rank adaptation, on two problems:

1. ATARI-game video sequences
2. Time-dependent PDE solutions of self-oscillating gels

ATARI frames

60 game sequences with varying number of frames. Each sequence is a 5-way tensor with shape $30 \times 28 \times 40 \times 3 \times N(i)$

Figure 2: Example MsPacman frame[3]



Figures 3 and 4: Number of frames vs execution time (left) and compression ratio (right) with $\varepsilon_{des} = 0.1$ [5]

We achieve 88% reduction in time with TT-ICE*!

Self-oscillating gel simulations

Parametric PDE simulated for 6400 unique parameter combinations. Each simulation is a 5-way tensor with shape $7 \times 13 \times 37 \times 3 \times 10$.

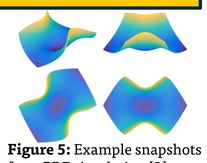
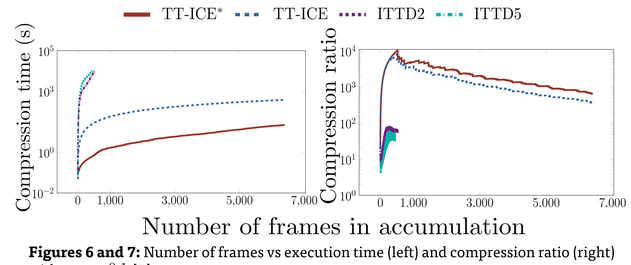


Figure 5: Example snapshots from PDE simulation [3]



Figures 6 and 7: Number of frames vs execution time (left) and compression ratio (right) with $\varepsilon_{des} = 0.1$ [5]

Low memory footprint of TT-ICE enables compression where literature fails!

References & Acknowledgements

[1] Oseledets, Ivan V. "Tensor-train decomposition." *SIAM Journal on Scientific Computing* 33.5 (2011): 2295-2317.
[2] Panagakis, Yannis, et al. "Tensor methods in computer vision and deep learning." *Proceedings of the IEEE* 109.5 (2021): 863-890.
[3] Liu, Huazhong, et al. "An incremental tensor-train decomposition for cyber-physical-social big data." *IEEE Transactions on Big Data* 7.2 (2018): 341-354.
[4] Le Trung, Thanh, et al. "Adaptive Algorithms for Tracking Tensor-Train Decomposition of Streaming Tensors." (2021).
[5] Aksoy, Doruk, et al. "An Incremental Tensor Train Decomposition Algorithm." *arXiv preprint arXiv:2211.12487* (2022).

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