

Seven-Loop Ultraviolet Behavior of Maximal Supergravity

PROBLEM

In four-dimensional $\mathcal{N} = 8$ supergravity, is the local seven-loop four-graviton $D^8 R^4$ ultraviolet coefficient c_7 zero or nonzero? In a specified dimensional-regularization and infrared-subtraction scheme, define

$$\mathcal{H}_4 \equiv Z_{\text{IR}}^{-1} \mathcal{M}_4, \quad c_7 \equiv \text{Res}_{\epsilon=0} \mathcal{P}_{D^8 R^4} \mathcal{H}_4^{(7)}, \quad D = 4 - 2\epsilon,$$

where the coupling normalization, external states, dimensional continuation, Z_{IR} , and normalized four-point projector $\mathcal{P}_{D^8 R^4}$ are fixed before the computation.

ANSWER AND VERIFICATION

A nonzero result must determine c_7 exactly or rigorously bound it away from zero. A zero result requires an exact calculation or proof; finite-precision agreement with zero is insufficient. Provide the derivation and any code needed for independent checking.

WHAT IS KNOWN

Explicit calculations establish the four-graviton ultraviolet behavior through five loops.^[1,4] In four dimensions, symmetry arguments predict that the six-loop $D^6 R^4$ coefficient vanishes, assuming continuous $E_{7(7)}$ duality is non-anomalous; this has not been checked by a six-loop calculation.^[3,4] Reproducing known lower-loop results and resolving six loops are useful computational milestones.

SIGNIFICANCE

In four dimensions, $D^8 R^4$ is the first four-graviton counterterm candidate not excluded by known supersymmetry and $E_{7(7)}$ constraints.^[2,3] A nonzero c_7 would establish a seven-loop ultraviolet divergence. A zero would reveal a further cancellation, but would not establish all-loop finiteness.

WHY COMPUTING COULD HELP

The question concerns a specified theory and requires no new experimental data. AI could assist with symbolic calculations, integral reduction, and checks against known amplitudes.

COMPUTE AND TOKEN ESTIMATE

Undetermined. A defensible estimate requires a specified method and measured lower-loop benchmarks, including the cost of independent verification.

COMMUNITY INPUT

We invite corrections to the formulation, alternative approaches, resource estimates grounded in benchmarks, and independently checkable partial or complete results.

SOURCES

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^[1] Z. Bern et al., “Ultraviolet Properties of $\mathcal{N} = 8$ Supergravity at Five Loops,” [arXiv:1804.09311](#) (2018).

^[2] H. Elvang and M. Kiermaier, “Stringy KLT Relations, Global Symmetries, and $E_{7(7)}$ -Violation,” [arXiv:1007.4813](#) (2010).

^[3] N. Beisert et al., “ $E_{7(7)}$ Constraints on Counterterms in $\mathcal{N} = 8$ Supergravity,” [arXiv:1009.1643](#) (2010).

^[4] G. Bossard and K. S. Stelle, “The Ultraviolet Problem in Supergravity,” [arXiv:2503.01648](#) (2025).